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EXPERIMENTAL OBSERVATIONS OF THE DYNAMIC BEHAVIOR OF LIQUID FILLED SHELL

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Department of the Army Project No. 503-03-009

BALLISTIC RESEARCH LABORATORIES

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ABSTRACT

Experiments with a 20mm M56 shell, whose cylindrical cavity, fineness ratio 2.7, was filled with different amounts of various liquids, demonstrated the existence of dynamic instability for a broad range of loading conditions. It was found that, for water-like liquids, the nutational component of yaw is divergent when 40 percent to 90 percent of the cavity is filled. For mercury, the yaw is divergent from about 25 percent to 100 percent. The severity of instability or the rate of divergence of yaw increases with the increase in the specific gravity of the liquid.

Stewartson's theory applies only to the steady state when the liquid is spinning with the full spin of the shell. Under this condition the theory predicts, for a cylindrical cavity, at what loading conditions (per cent of fill) the liquid filled shell system will be unstable. Predicted instabilities are usually confined to relatively narrow zones of loading conditions. The observed, very broad zones of instabilities, are attributed to the transient state of liquid rotation. The time required to reach the steady state is relatively long, especially for liquids with viscosities less than, say, 10 centi-stokes (c.s.). During this time, if the shell is

unstable, the yaw may reach a prohibitively large level rendering such shell useless in practice. Baffles in the cavity effectively suppress the transition phase.

TABLE OF CONTENTS

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1. INTRODUCTION

It is a well-established fact that some spin stabilized shell if filled with a liquid become dynamically unstable. The problem is well illustrated by Plate 1. The cylindrical cavity of this particular 20mm shell was filled with mercury. The shell is gyroscopically stable with a stability factor of about 2.5. It began its flight with very small yaw but at 280 feet from the muzzle the yaw has increased to about 80°. Such exceptionally rapid divergence of yaw is usually associated with heavy liquids. Lighter liquids cause milder instabilities. Nevertheless, even milder instability persisting for a sufficiently long time may render the shell useless in practice.

Although the problem of stability of liquid filled shell has been under investigation for a long time, unfortunately, to date, no infallible general principle applicable to all cases has emerged.

Largely, through experience supplemented, however, by theoretical analyses, the following variables appear to influence the dynamics of liquid filled shell:

1. Air space in cavity or per cent of filled volume.
2. Geometry of the cavity.
3. Specific gravity of the liquid.
4. Liquid viscosity.
5. Spin level of the shell.
6. Shell velocity.
7. Quadrant elevation.

It is not yet possible to specify the relative importance of these variables in general. However, for a cylindrical cavity, the air space and the geometry of the cavity are probably the most important, followed, perhaps, by the specific gravity of the liquid. The viscosity has at least two effects. First, higher viscosity expedites the attainment of full spin and, hence, shortens the duration of the transitional regime which, as we shall see later, is very important. Second, higher viscosity probably inhibits, within the disturbed fluid, the development of oscillations of

large amplitude and causes these disturbances to die out faster. The spin level of the liquid, rotating as a rigid body, controls the frequencies of its free oscillations. Since instability of the shell occurs only whenever one of the liquid frequency is in resonance within the nutational frequency of the shell, the spin of the shell is important. It can be shown (see W. E. Scott's forthcoming BRL report) that the frequencies of free oscillations of the liquid are given by

$$\frac{\omega_{i,j}}{\Omega_s} = f_{i,j} \quad \frac{\Omega_L}{\Omega_s} \cdot$$

where Ω_L is the spin of the liquid, Ω_s is that of the shell, and $f_{i,j}$ is a certain function which defines the double infinity of frequencies of the liquid. The nutational frequency of the shell is given by

$$\frac{\dot{\phi}_1}{\Omega_s} = \frac{A}{2B} \quad (1 + \quad \sigma)$$

where A, B are the polar and transverse moments of inertia of the shell and

$$\sigma = \sqrt{1 - \frac{1}{s}}$$

where s is the gyroscopic stability factor. This factor varies as $\frac{\Omega_s^2}{C_{m_\alpha}}$.

The C_{m_α}, the overturning moment coefficient, depends, among other things, on Mach number or velocity. Thus, the spin and the velocity change σ and, hence, the nutational frequency. Theoretically, therefore, it is possible, with the liquid spinning with the full spin or the shell $\frac{\Omega_L}{\Omega_s} = 1$, to alter the resonance conditions for a given shell (A, B) by changing σ either by spin or velocity (through change in C_{m_α}) or both. The effect of quadrant elevation for high angle fire may well be associated with changing relationship between the $\frac{\omega_{i,j}}{\Omega_s}$ of the liquid and the nutational frequency of the shell.

In the history of studies of this problem, two milestones should be briefly mentioned. In 1940, E. A. Milne[1] made a thorough theoretical analysis of this problem on the basis of a mathematical model, developed earlier by others, of the stability of a shell containing a spheroidal

cavity completely filled with inviscid fluid, rotating with the full spin of the shell. The stability of such a system depends on the geometry of the cavity, the major and minor axes of the spheroidal cavity, the mass of the liquid, and the inertial properties of the shell. He showed that, for this model, the region of instability was defined by the following inequalities:

$$x > y, \quad x - y < 1 < x + y$$

where

$$x = \sqrt{\frac{K^2 - 1}{K^2 + 1} \quad \frac{B}{A}} \qquad , \; y = \frac{2^K}{K^2 + 1} \sqrt{\frac{2}{5} \; \frac{M \;\, a^2}{A}}$$

where K = fineness ratio of the cavity, $\frac{\ell}{2a}$

M = mass of the liquid

B, A = transverse and axial moments of inertia of the shell.

When all the firings from 1926 onward were analyzed and each shell was marked, according to its behavior, as either S = stable or U = unstable, and plotted in the (x, y) plane, it was found that a fairly well-defined curve could be drawn, in this plane, separating the S from the U points. This curve, however, appeared to be only tangent to the theoretical instability boundary $x - y = 1$ near $y = 0$ and was purely empirical elsewhere. Nevertheless, it proved to be very useful in practice.

Milne further explored the effect of viscosity and of air space and suggested how these parameters would influence the stability. However, such effects could not be adduced quantitatively from the available theory.

The next high light in this story is the work of K. Stewartson[2, 3]. He extended the analysis to a cylindrical cavity partially filled with an inviscid fluid rotating as a rigid body with the full spin of the shell. This model, of course, is much more realistic than the completely filled spheroidal cavity but, also, is considerably more complicated. Stewartson showed that instability results only when the poles of a certain very complicated nondimensional function, defined entirely by the dimensions of the cavity and the fluid mass, occur close to the nutational frequency of the shell. In certain firing trials at Porton (British Proving Ground), this theory was tested and

appeared to be fully confirmed. For different fineness ratios of the cylindrical cavities and different air spaces, all shell predicted to be unstable gave short ranges and all shell predicted to be stable gave full range. This problem, therefore, appeared to have been solved.

Recently, however, at the Aberdeen Proving Ground, in the course of development of a certain weapon system, we have encountered a severe case of dynamic instability of liquid filled shell. For the geometry of its cylindrical cavity and for various air spaces which have been fired, it should have been stable according to Stewartson's analysis, but it wasn't. It was violently unstable under all conditions of test firings.

Further search of the literature proved not very helpful. A very extensive amount of both theoretical and experimental work on sloshing of fluids in tanks, in connection with the development of liquid fueled rockets, has been done in recent years. There is a great variety of theoretical treatments of related problems but mostly, however, with nonspinning fluids. A fairly extensive bibliography of papers in this field is to be found in reference (4).

Therefore, it appeared desirable to look into this problem further. Since detailed experimental data on the dynamics of liquid filled shell are practically nonexistent, it appeared particularly desirable to obtain such information. Availability of the spark ranges at BRL made the acquisition of such data feasible.

On the theoretical side, we set out to examine, within the framework of modern ballistic theory, four fundamental cases. In the order of increasing complexity, these are:

1. Nonspinning liquid
    a. Completely filled cylindrical cavity
    b. Partially filled cylindrical cavity
2. Spinning liquid
    a. Completely filled cylindrical cavity
    b. Partially filled cylindrical cavity

Problems 1a and 1b have been solved and the reports published by Scott, references (5) and (6). While some of the conclusions are not new, Scott's independent formulation and analysis of these problems show much more clearly than we were able to find in the available literature, the effect of the liquid on the yawing motion of the shell. His work is particularly valuable for the comparison of the theoretical predictions with the observed behavior of the shell in flight. The available treatments of these problems require a considerable amount of adaptation, and are also either too restrictive or too general to be directly useful.

Scott's analysis shows that for a completely filled cavity, the only effect the liquid produces on the dynamics of the shell is to modify its inertial properties. For a partially filled cavity (nonspinning liquid), the liquid, in addition to altering the inertial properties of the empty shell, affects its yawing frequencies. There is no contribution by the liquid to the damping of the system. Since nonspinning liquid can, in reality, occur only in nonspinning shell, the conclusion of this analysis is that the liquid filler has no detrimental effect on the dynamics of fin stabilized shell. This conclusion has been verified by a limited number of experiments[6].

Problems 2a and 2b have been solved by Stewartson. Scott, however, has removed Stewartson's assumption of small liquid mass. For heavier liquids, this modifies the nutational frequency of the system and, hence, changes Stewartson's stability regions. Moreover, by carrying, in the analysis, the ratio of the spin of the liquid to that of the shell (in Stewartson's analysis, this ratio is unity) and choosing a different boundary condition which is to be made homogeneous, Scott's equations are more physically understandable. They show clearly how the spin of the liquid modifies the nonspinning case. The analysis of problem 2a is being published and that of 2b will be published shortly. The present report deals only with the experimental results.

2. EXPERIMENTS

The test vehicle selected was the 20mm M56 (T 282) spin stabilized shell. It was cheap, abundant, and of convenient size for the work in the spark range. Its schematic drawing is given in Figure 1. The normal cavity of this shell has thicker walls under the rotating band because this shell is designed to be fired at high velocity. The cavity, therefore, was modified to make it cylindrical with a fineness ratio 2.68. Most of the firings were done from a gun with a twist of rifling of one turn in 25 calibers of travel with a muzzle velocity of about 2700 fps. The shell were fired thru a spark range. This range is fully described in reference (7). Briefly, it contains 45 spark stations over a base line of 285 feet. The first observing station is 15 feet from the muzzle. The observed trajectory, therefore, is about 4400 calibers long over which the 20mm shell executes about 13 yawing periods. The accuracy with which the shell position and attitude can be measured on photographs is 0.01 inches, and for these relatively short shell, 5 minutes of arc respectively. If dissimilar pins are inserted in the base of the shell, their orientation at each station can be determined to an accuracy of about 2 degrees. These pins are used to measure the axial spin of the shell. Thus the range data provide a fairly accurate history of the yaw, the axial spin and the velocity of the shell over the initial 4400 calibers of travel. Three pieces of data, therefore, are directly obtainable from the photographic plates: (a) position of the shell at a given time, (b) the attitude of the shell axis, or the yaw, and (c) the orientation angle of the base pins vector.

Analysis of these data is handled in the usual manner which is fully described in reference (8). The analysis of yaw, for example, is done by fitting an epicycle to the observed components of yaw. With rigid filler, an epicycle fits the yawing motion of this shell very well[9]. With the liquid filled shell, however, there is no a priori assurance that an epicyclic fit could be made. This is so for several reasons.

For the solution of the differential equation of the yawing motion, applicable to the data in the spark range, it is assumed that over the relatively short time of flight, all the aerodyanmic coefficients are constants. The only dependent variable which is permitted to vary slowly is the nondimensional axial spin. Its slow variation is taken care of by a perturbation technique. However, for liquid filled shell, the axial spin varies much more rapidly, at least initially, than is the case for a rigid shell. Moreover, the liquid produces additional forces and moments which may not be constants. Both of these effects may change the epicyclic character of the yawing motion.

To test the effect of the observed variation of axial spin on the yawing motion, analogue computer runs were made for two liquids: water and mercury. In spite of the considerably faster initial decay of the axial spin with water, than would be the case if the filler were rigid, no significant effect on the character of the epicycle could be detected. With mercury, the story is different. Because of the extremely rapid initial decay of the axial spin in this case, the yawing motion becomes divergent but finally settles down to a normal damped motion by mid-range. It should be kept in mind that these investigations pertain only to the effect on the yawing motion of postulated variable axial spin and have nothing to do with possible other effects the liquid may or may not have on such a motion.

It was pleasantly surprising, therefore, that for all liquids provided that the yaw was not excessive, i.e. $\delta \leq 10^{\circ}$, the observed yawing motion could be represented quite well by the usual epicycle of the form[10].

$$\lambda = K_{10} \exp\left(-\alpha_1 + i\phi'_1\right)p + k_{20} \exp\left(-\alpha_2 + i\phi_2'\right)p$$

where λ complex yaw

$\alpha_{1,2}$ are the nutational and precessional yaw damping rates respectively

$\phi'_{1,2}$ are the circular rates

p the distance along the trajectory in calibers.

For a rigid shell, the yaw damping rates are given by the following expressions:

$$\alpha_{1,2} = 1/2 \left[\left(1 \pm \frac{1}{\sigma}\right) H \mp \frac{2}{\sigma} T + \frac{(1 \pm \sigma)}{\sigma^2} D \right]$$

where

$$H = \frac{\rho S d}{2m} \left[C_{L_\alpha} - C_D - k_t^{-2} \left(C_{M_q} + C_{M_{\dot\alpha}} \right) \right]$$

$$T = \frac{\rho S d}{2m} \left[C_{L_\alpha} + k_a^{-2} C_{M_{p\alpha}} \right]$$

$$D = \frac{\rho S d}{2m} \left[C_D - k_a^{-2} C_{\ell_p} \right]$$

$$\sigma = \sqrt{1 - \frac{1}{S}} \qquad S = \frac{\nu^2}{4M} \quad \text{gyroscopic stability factor}$$

$$M = \frac{\rho S d}{2m} k_t^{-2} C_{M_\alpha} \qquad k_a^{-2} = \frac{md^2}{A}, \quad k_t^{-2} = \frac{md^2}{B}$$

Also

$$\phi'_{1,2} = 1/2 \, \bar\nu \, (1 \pm \sigma)$$

$$\bar\nu = \frac{A}{B}\nu = \frac{A}{B} \frac{\omega_1 d}{u_1}$$

The accuracy of the epicyclic representation of the observational data varies somewhat depending on the magnitude of the yaw and the nature of the liquid. For water, for example, the least square fit of an epicycle to the observational data is characterized by the standard deviation of about 7 minutes of arc. This is comparable with the accuracy of the data and, hence, can be considered as satisfactory. A more visual sense of the goodness of fit can be obtained from the plots on semi-log paper of the amplitudes of each component of yaw against distance. According to the linearized theory, the logarithmic decrement of the amplitude of yaw should be a constant, i.e., on semi-log paper the plot of the amplitude vs distance should be representable by a straight line. Several such plots are reproduced in Figure 2. The plotted points are from a preliminary hand reduction prior to the machine reduction and the application of differential corrections. On the whole, straight lines do fit the data reasonably well.

One can now go further and complete the analysis of the yawing motion by computing what one may call "effective aerodynamic coefficients", i.e., from the observed yaw damping rates compute H, T, and D. It is to be understood, of course, that for a liquid filled shell, these are not the true aerodynamic coefficients but are simply certain parameters of the epicycle. Only if it can be demonstrated that their numerical values are the same as for an equivalent rigid shell can they be called aerodynamic coefficients. Then the inference can be drawn that the liquid does not contribute measurable amounts to corresponding aerodyanmic forces and moments.

3. THE EFFECT OF WATER

The water, as a filler, was tested most extensively. The "effective aerodynamic coefficients" and the circular frequencies, for this liquid, are given in Figures 3 and 4. The scatter in the lift coefficient is to be expected because it is determined from the swerving motion which method is less accurate than that using two center of mass positions. The scatter in the other coefficients is somewhat larger than one would expect if the shell was rigid. This might reflect the fact that the fitted epicycle is a fairly coarse approximation to the actual yawing motion. Also, there appears to be a systematic difference in the observed "coefficients" between lower and higher percentages of fill. To test the significance of this difference, the data were divided into three groups. The first group consisted of all rounds from 0 - 35 percent of fill; the second from 50 to 90 percent of fill and the third group consisted of rounds of 100 percent fill.

The following table gives the average values for each group together with its standard deviation and the significance of the difference of 100 percent and 50 - 90 percent groups relative to the 0 - 35 percent group.

For this particular shell and this particular liquid, the principal effect of the liquid filler seems to be a decrease in the effectiveness of the yaw damping moment, $C_{M_q} + C_{M_{\dot{\alpha}}}$. Other effects are relatively minor.

The yaw damping rates are the logarithmic decrements of the amplitudes of the precessional and nutational arms of the epicycle. In ballistic practice, these are defined in such a way that a positive yaw damping rate corresponds to shrinking of the amplitude and a negative rate to its growth. Thus, a dynamically stable shell should have a positive yaw damping rate for each component of yaw.

Figure 5 shows the yaw damping rate for water as the liquid filler for various percentages of fill. As was expected, the liquid affects only the nutational yaw damping rate. The precessional component is

TABLE I

Liquid (H_2O) Filled Shell

Average "Aerodyanmic Coefficients"

Coefficients	Group I 0 - 35%	Group III 100%	Significance	Group II 50 - 90%	Significance
C_{M_α}	.0428 ± .0009	.0431 ± .0011	n.s.*	.0417 ± .0014	Signif. on 3% level
C_{L_α}	.0424 ± .0016	.0439 ± .0024	n.s.	.0428 ± .0023	n.s.
$C_{M_{p\alpha}}$	.0037 ± .0010	.0029 ± .0019	n.s.	.0013 ± .0021	s.
$C_{M_q} + C_{M_{\dot\alpha}}$	-.110 ± .011	-.079 ± .008	v.s.*	+.029 ± .046	v.s.
ϕ'_1	2.306 ± 041	2.302 ± .047	n.s.	2.260 ± .052	Signif. on 2%
ϕ'_2	.187 ± .006	.192 ± .004	n.s.	.191 ± .006	n.s.

* n.s. = not significant
v.s. = very significant

practically unaffected. What was unexpected is a broad zone of fill conditions, from about 40 percent to 90 percent, for which this component of yaw is divergent. The shell, therefore, for these conditions of fill, is dynamically unstable. There is a curious scatter around 40 - 50 percent of fill which remains unexplained.

It is worthy of note that Stewartson's theory predicts instability, for this case, in a narrow zone of 43 percent to 48 percent of fill. This is the only zone of instability predicted. The fact that at 50 percent of fill the divergence of yaw is very mild suggests that the fluid may be approaching a condition of spin at which Stewartson's theory may begin to be applicable. With full spin of the liquid at 50 percent fill the shell should be stable. Since our fill conditions were not precisely controlled, in that they were predicated on the average size cavity, it might well be that the four discordant points actually lie in the above zone, 43 - 48 percent, although nominally were designated as 40 percent and 50 percent respectively. If this is so, then the discordant points, showing larger divergence of yaw than those at 50 percent fill, might be considered as a suggestive confirmation of theoretical prediction.

4. THE EFFECT OF OTHER LIQUIDS

In addition to water, several other liquids were used as fillers in a limited firing program. These liquids were:

Liquid	Sp. gr.	ν c.s.
Mercury (Hg)	13.5	0.1
Tetrabromoethane ($CHBr_2$)	3.0	3.4
Glycerin	1.26	1000

The yaw damping rates for these liquids are given in Figure 6. Average values for water are included for comparison.

Several interesting features are to be noted in this figure. The severity of instability increases with an increase in the specific gravity of the liquid. Thus, at 70 percent of fill, where the growth of the nutational component of yaw appears to be most severe, for water, this component of yaw will double in about 3460 calibers of travel (226 feet), for $CHBr_2$ in 1150 calibers, and for mercury, in about 350 calibers. The pattern of instability versus percent of fill remains, however, essentially similar as if the water curve was pulled down roughly in proportion to the ratio of the specific gravities of the liquids concerned.

The analysis of mercury filled rounds, because of very rapid divergence of yaw, is less certain. That is why the symbols in Figure 6 are placed in parentheses. It is to be recalled that analogue computer run for a completely mercury filled cavity showed an initial divergence of yaw due to very rapid spin decay. This divergence, however, is very mild in comparison with the observed divergence for a 100 percent full case. The former rate of growth of yaw is less than 0.1×10^{-4} per caliber of travel; the observed divergence is 10×10^{-4} or one hundred times greater. Therefore, within the accuracy of the analysis of these rounds, the effect of the spin decay on the yawing motion of the shell has been neglected.

The similarity between the three curves may be somewhat fortuitous, occurring only at very early stages of flight when the spins of these three liquids are far removed from the full spin condition. Nonspinning

liquid produces no detrimental effect on the dynamics of the shell. For the 70 percent fill, for example, fully spinning liquid also produces no detrimental effect as predicted by Stewartson and verified by our firings with glycerin. Therefore, instability for this loading condition commences at some condition of fluid motion, be it spin or secondary flows or both, and gradually ceases at a spin distribution less than full, because theoretically it requires a relatively long time for the liquid to reach full spin. The yaw damping curves for the three liquids, H_2O, $CHBr_2$, and Hg, Figure 6, correspond, therefore, because of their different viscosities, to somewhat different states of fluid motion. At later times, the observed relationship between their yaw damping rates will be altered.

Glycerin, because of its very high viscosity, attains rigid body rotation very rapidly, in the order of 20 milliseconds of flight. A considerable fraction of the full spin is attained while the shell is in the gun. Hence, Stewartson's theory, for an inviscid fluid, may conceivably apply to this case. As previously mentioned, Stewartson predicts instabilities only between 43 percent and 48 percent of fill and nowhere else. The shell have been tested at 90 percent, 70 percent and 48 percent fill. As Figure 6 shows, the shell is unstable only at 48 percent. This, therefore, may be considered as a verification of Stewartson's theory.

5. AXIAL SPIN

As previously mentioned, dissimilar pins in the base of the shell permit determination of its axial spin history in the range. Some typical spin histories for an empty shell, 100 percent filled with water, glycol, thetrabromoethane, and mercury are shown in Figure 7. It is seen that the spin decay of an empty shell is very modest; it decreases in the first 120 milliseconds of flight by about 1.6 percent. If the cavity was filled with a rigid substance, inertially equivalent to water, the spin would decrease only 1.5 percent. However, with water it decreases by 5.6 percent, with thetrabromoethane by 15 percent, and with mercury, in the first 60 milliseconds of travel, by 25 percent. However, even with mercury the shell remained gyroscopically stable in spite of a substantial loss of its axial angular momentum.

The relatively rapid spin decay of the liquid filled shell is due, of course, to a continuous transfer of shell angular momentum to the liquid. The two are coupled by the friction at the walls of the cavity and the radial velocity gradient, thus established, transmits angular momentum to the liquid. The efficiency of this process depends on whether the momentum is transferred by laminar or the turbulent shearing stresses at the wall. The latter is many times more efficient than the former. However, initially, the boundary layer is probably laminar because of the relatively low Reynolds number. If one defines the Reynolds number as

$$Re = \frac{(V_o - V)\delta}{\nu}$$

where V_o is peripheral velocity of the shell

V is the velocity at the edge of the boundary layer ($V = 0$ initially) and δ is the thickness of the boundary layer, then initially Re is small because of small δ. It increases to its maximum value

$$Re_{max} = \frac{V_o a}{\nu}$$

and then decreases because of the decrease in the difference between ω_o and ω as the liquid approaches rigid body rotation; i.e.,

$$Re \rightarrow \frac{(\omega_o - \omega)a^2}{\nu}$$

For the condition of our experiments with water, the maximum Re is about one-half million. This value is reached in about two seconds of time of flight. In the range, therefore, where observations extend only to 0.1 seconds, the maximum Reynolds number is of the order of 30,000. Schlichting[12] has studied the stability of the laminar boundary layer inside a spinning cylinder whose spin is started impulsively from rest, a situation not dissimilar to ours. He has shown that subject to small disturbances, the laminar boundary layer is stable up to Reynolds number of 66,000, which is the minimum critical Reynolds number. Therefore, in our case, the transition to turbulence is not likely to occur during the observed flight of 0.1 seconds.

In view of these considerations, it was assumed that the fluid motion can be represented by a two dimensional laminar flow. A theoretical analysis of this problem was made by Scott, reference 11. Figure 8 shows that for water, the two dimensional laminar flow hypothesis is inadequate.

The observed spin decay can be represented as a product of two exponentials

$$\omega = \omega_o e^{-\beta x} e^{-f}$$

where the first exponential is due to the air torque and the second due to the liquid.

Using the usual laminar boundary layer approximations, it can be shown that laminar shear stress on the walls of the rotating cylinder, started impulsively from rest, is

$$\tau_W = \rho \frac{a\omega}{\sqrt{\pi}} \sqrt{\frac{\nu}{t}}$$

where ω is the spin of the cylinder, a is radius of the cavity and ρ, ν are the density of the liquid and its kinematic viscosity respectively. The torque due to the liquid, therefore, is

$$T_\ell = a \cdot \tau_W \cdot S$$

where S is wetted area. The differential equation governing the spin decay of the shell can be written as

$$A\dot{\omega} = -T_{air} - T_{liquid}$$

$$= -\rho_a \frac{d^2}{2} S U \omega C_{\ell_p} - T_{\ell}$$

where A is the polar moment of inertia of empty shell. From this, it follows that

$$f = \int_0^t \frac{T_{\ell}}{A\omega}\, dt$$

and $$\beta = \frac{\rho_a d^2 S C_{\ell_p}}{2A}$$

For the laminar boundary layer, using the expression for the liquid torque and performing the integration, the exponent "f" can be written as

$$f = C_1 \sqrt{t}$$

where C_1 is a computable constant. This expression fits the laminar curve of Figure 8, which was computed by a different method in reference 11, very well. This is not surprising. The thickness of the laminar boundary layer, at the end of the observed flight, is only about 10 percent of the radius of the cavity. Therefore, the boundary layer solution is a reasonable approximation to the more exact solution. At later times, the two solutions should become progressively divergent.

For the turbulent boundary layer, the unsteady solution is unknown. However, in order to get some idea of what the spin decay would be if the boundary layer was turbulent, Dr. Sternberg suggested the use of the momentum integral with an assumed velocity distribution. The turbulent shear stress at the wall is

$$\tau_W = \frac{d}{dt}\int_0^{\delta} \rho \; U \; d \; y \tag{1}$$

if one assumes the velocity distribution

$$\frac{U}{U_o} = (1 - \frac{y}{\delta})^{1/7} \tag{2}$$

the shear stress becomes

$$\tau_W = \frac{7}{8}\rho \; U_o \; \frac{d\delta}{dt} \tag{3}$$

The torque $T_{\ell} = a. \; S \; . \; \tau_W$ (4)

and the "f" value

$$f = \int_0^t \frac{T_{\ell}}{A\omega} dt = \int_0^t \frac{aS\tau_W}{A\omega} dt = aS \; \frac{7}{8} \; \frac{\rho U_o \delta(t)}{A\omega} \tag{5}$$

if ω is assumed constant.

To get $\delta(t)$ we proceed as follows:

$$\tau_W = C_f \; \frac{1}{2}\rho \; U_o^{\;2} = \frac{7}{8}\rho \; U_o \frac{d\delta}{dt} \tag{6}$$

Using flat plate results:

$$C_f = \frac{.0577}{(R_e)^{1/5}} \tag{7}$$

and

$$\delta = \frac{.37 \; x}{(R_e)^{1/5}} \tag{8}$$

and eliminating $\underline{x}$ between eqs (7) and (8) one obtains $C_f = g(\delta)$. Substituting in eq (6) and integrating one gets

$$\delta(t) = .0639 \; (\frac{\nu}{U_o})^{1/5} \; U_o^{\;4/5} \; U_o^{\;4/5} \; t^{\;4/5} \tag{9}$$

Finally, substituting (9) into (5) one gets the desired result. The turbulent case of Figure 8 was computed in this manner. The accuracy of these approximations is unknown but the results should be adequate for illustrative purposes. It may be of interest to note that if one used the flat plate local turbulent skin friction coefficient directly,

$$\tau_W = C_f \quad q = \frac{.0577}{R_e^{1/5}} \cdot q$$

and substituting for $x = U_o t$ in R_e, one obtains a similar expression for "f" of the form

$$f = c\ t^{4/5}$$

Only the numerical factor is 1.55 times smaller than the numerical factor obtained by the momentum method. Figure 8 shows that neither the laminar nor turbulent boundary layers produce the type of torque which is necessary to account for the observed spin decay. The observed curve lies in between these two extremes. Nevertheless, the observed spin decay can better be represented by the $\sqrt{t}$ law suggesting, thereby, that the boundary layer is laminar.

Scott has found that it is possible to represent the observed spin decays of various liquids, with the two dimensional laminar hypothesis, by using fictitious viscosities. Thus for water, an "effective" viscosity 32 times greater than its natural viscosity, gives an excellent representation of the observed spin decay; for glycol, however, its natural viscosity suffices, Figure 9. The following fictitious viscosities represent the observed spin decays of various liquids:

Liquid	ν_o c.s.	ν/ν_o
Hg	0.11	290
H_2O	1.	32
$CHBr_2$	3.4	14
Glycol	660	1

If the boundary layer is not turbulent, and it does not appear to be, then the "effective" high viscosities must be accounted for by some mechanism which increases the laminar shear stress at the wall. Such mechanism may be provided by the secondary flows in the cavity. It is well known that in an inclosed cylinder filled with a liquid and started to spin impulsively, secondary flow develops as a result of end effects[13]. Such circulation may well steepen the velocity gradient at the wall and, hence, effectively increase the laminar shear stress.

In order to examine this hypothesis in greater detail, an "impulsive spin generator" was designed and built[14]. It consists of a transparent cylinder, geometrically homologous to the shell cavity, mounted on a rotating table which could be instantaneously engaged to a rotating fly wheel operated by an air turbine. Two magnetic pickup coils measure the speed of rotation. If spun at 7000 rpm, the apparatus correctly simulates the ratio of centrifugal to deceleration forces of the shell in free flight.

Various experiments were performed in an attempt to detect the presence of a turbulent boundary layer. Tufts were unsuccessful. The use of "aluminum lining" powder, flaky bits of aluminum which, in suspension, align themselves in the direction of shear surfaces, is a well-known technique for visual detection of turbulence. The results were negative; the flow appeared to be laminar.

To detect and measure the secondary flows, a method was devised employing a homogeneous suspension of particles in the fluid. The particles were black ethyl cellulose ($\rho = 1.11$) suspended in an aqueous solution of gylcerin, the solution having a kinematic viscosity of .062 Stokes. Their motion was photographed with a high-speed camera. Fuller account of these experiments will be given elsewhere. Here, only a few pertinent results will be mentioned.

When the film was projected, the existence of secondary circulation was at once apparent. The fluid in the cylinder divided itself into upper and lower cells. In each cell, a doughnut-like vortex was generated,

carrying fluid particles from the periphery toward the axis and up, in the upper part of the cylinder, and down, in the lower part. Figure 10 shows the trajectories of some of the particles. These show clearly the two-cell structure of the fluid motion and a strong indication of a development of a doughnut-like vortex in each cell.

Distribution of vorticity throughout the fluid, at two instances of time, is shown in Figure 11. Corresponding theoretical distribution for laminar flow, no end effects, and without the secondary flows, is given for comparison.

In the solution of the two-dimensional laminar flow problem[11], viscosity and time appear only as a product, νt. Therefore, the observed distributions of vorticity, at various peripheral speeds, can be roughly approximated by the theoretical distributions either with fictitious viscosities and real times or natural viscosity and later times. The following tables give the ratios of the fictitious to natural viscosity for various cases. With the fictitious viscosity, theoretical curves could be approximately fitted to the observed points.

TABLE II

Matching the observed distributions of vorticity by fictitious viscosities and real times.

	ν/ν_o		
Observed Time, t sec.	$\omega_o = 471$	1434	3065 rpm
1	1	1.5	2
2	1.5	2	3
3	1.7	2.3	4
4		2.5	

The results show that the greater is the peripheral speed, the higher is the "effective" viscosity and, hence, the effective shear stress at the wall. However, the details of the mechanics whereby the secondary circulation, of the observed nature, increases the laminar shear at the wall, are not clear.

If the trend of the increase in the effective viscosity with the increase in the peripheral speed, as shown by table experiments, continues, then one would expect considerably higher ratios of ν/ν_o for the 20mm shell whose spin is about 10^5 rpm. The following empirical relationship seems to tie the 20mm shell and the rotating cylinder results in a reasonable way, although its physical significance, if any, is not at all clear. The ratio of the effective viscosity to the natural viscosity for various liquids and spins can be represented by the following relationship:

$$\frac{\nu}{\nu_o} = \left(1 + \frac{31}{\nu_o}\right)\left(\frac{v}{v_r}\right)^{1/3}$$

where ν_o viscosity of the liquid in centi-stokes, and v and v_r are the peripheral velocities of the rotating cylinder (table experiments) and the 20mm shell respectively. A comparison between the "observed" values and the empirical formula above is shown in the following table:

20mm Shell				
Liquid	V_r cm/sec.	ν_o c.s.	ν/ν_o obs.	ν/ν_o emp.
Hg	7.6×10^3	0.11	290	290
H_2O	7.6×10^3	1.0	32	32
$CHBr_2$	7.6×10^3	3.4	14	10
Glycol	7.6 x 103	660	1	1
Rotating Cylinder	v cm/sec.	ν_o c.s.	ν/ν_o obs.	ν/ν_o emp.
H_2O + Glycerin	603.4	6.2	3.0	2.6
H_2O + Glycerin	282.3	6.2	1.9	2.0
H_2O + Glycerin	92.7	6.2	1.4	1.4

The agreement is not bad and the empirical formula can, perhaps, be used to estimate an early history of the shell's axial spin decay.

However, even with the enhanced, fictitious viscosity, it takes a relatively long time for a low viscosity fluid, such as water, to acquire full spin and reach a state at which Stewartson's theory becomes applicable.

Meanwhile, if the shell is dynamically unstable during the transition to the full spin regime, the yaw may become prohibitively large rendering the shell useless in practice. To test the effective duration of the transient regime and its effect on the theoretically predicted stable flight (when the liquid is fully spinning), the 20mm shell were fired at longer ranges.

6. LONG RANGE FIRINGS

Six groups of 20mm shell were prepared, five shells in each group, for firing at a vertical target located 600 yards from the muzzle. The shell, identical with those fired in the spark range, were loaded with various percentages of fill with two types of liquids: water and thetrabromoethane, in accordance with the following schedule.

Groups	H_2O (Sp. Gr. = 1) Fill, percent	Groups	$CHBr_2$ (Sp. Gr. = 3.0) Fill, percent
1	100	4	90
2	70	5	70
3	50	6	50

The shells were to be fired from a standard gun (twist of rifling 1:25) at a muzzle velocity of 2700 fps. However, due to an erroneous loading, the muzzle velocity turned out to be closer to 2600 fps than to 2700 fps. This small difference in the muzzle velocity is unimportant for the purposes at hand.

The experiment consisted of measuring the velocity by two velocity screens 42 apart, with the first screen located 26' from the muzzle, and the usual time of flight screen at 600 yards. A paper screen on the second velocity screen gave some indication of the initial yaw. All initial yaws appeared to be small.

The analysis of the firings consisted in the evaluation of an "effective" ballistic coefficient over about one second of the time of flight.

The results are given in the following table:

TABLE III

Ballistic coefficients of water filled 20mm M56 shell obtained from time of flight firings to 600 yards.

Percent of Fill	Weight of Shell Grains	C	C - Expected
100	1128	.274	.275
	1131	.278	
	1129	.277	
	1129	.273	
70	1087	.216	.266
	1089	.218	
	1087	.252	
	1090	.270	
50	1088	.266	.265
	1100	.256	
	1080	.259	
	1077	.260	
	1092	.238	

The average cavity requires 7.8 milliliters to fill, corresponding to a weight of water of about 120 grains. Apparent inconsistencies in the above weights arose from variations in the weights of the empty shell.

What was the expected behavior of these shell as inferred from the spark range results, illustrated in Figure 5? The 100 percent filled shell are stable and, hence, should fly well. The consistency of the ballistic coefficients and their agreement with the expected value verifies this. According to Figure 5 the shell, in the transient regime, with 70 percent water filled cavity, is unstable. The degree of instability is such that the yaw doubles every 3460 calibers of travel. Since the distance to the 600 yard target is about 28,000 calibers if this divergence of yaw

persisted throughout the flight, the yaw on the target would be about 2^7 times as great as the initial yaw. Even if the initial yaw is of the order of one degree or so, the final yaw would be prohibitively large. However, if the "full" spin is reached at, say, one-third of the range, then the shell will become stable, as was demonstrated by our firings with glycerin, and from this point on the yaw will decay. If this history of yaw is followed, on would expect that the average ballistic coefficient will be decreased by about 1.6 δ_o^2 percent where δ_o, the initial yaw, is in degrees. The 50 percent filled shell were also expected to show lower ballistic coefficient. The divergence of yaw, at this fill, is milder than at 70 percent and the liquid should reach "full" rotation sooner. However, the behavior of shell at this percentage of fill is on the ragged edge because at 48 percent fill, the shell is predicted to be unstable as, again, was verified by the spark range firings with the glycerin. Therefore, in a group of shell fired with this percent of fill, there might be an occasional "maverick" due to inadequately controlled loading conditions. One of the rounds in the 50 percent fill group appears to be in this category.

The story of firings with thetrabromoethane is short. Not a single shell of fifteen fired hit the target in spite of the valiant efforts to do so. All fell short even when the gun was elevated to shoot 40 feet above the target.

The rate of divergence of the nutational component of yaw for $CHBr_2$ is about three times as large as for water. Therefore, at corresponding times in flight, the yaw levels for these rounds must have been considerably larger. This demonstrates, rather dramatically, the importance of the unstable transient regime for heavier liquids.

7. BAFFLED ROUNDS

The analysis of the axial spin of the shell shows that, during the observing period in the firing range, all fluids, with the exception of such high viscosity liquids as glycerin, $\nu = 1000$ c.s., or glycol, $\nu = 660$, continue to absorb axial angular momentum from the shell. Hence, they are in an unsteady state, only gradually reaching rigid body rotation. It is clear, therefore, that the dynamic instability of the nutational component for a broad range of fill conditions is somehow associated with this transient phase of liquid motion. The mathematics of this problem is extremely complicated. Certain tentative approaches are being made and the solution may eventually be found.

However, from the engineering point of view, it would appear that this transient unstable phase can be eliminated by forcing the liquid to acquire full spin at the muzzle. This, of course, can be done by inserting baffles into the cavity. It is a crude approach but it was tried to test the expectations. The baffles consisted of two cross plates extending from the base of the cavity to the base of the fuze. The following fillers were fired with baffles with the following dynamical results:

Filler	Per Cent	Motion
$CHBr_2$	70	Stable
	50	Unstable
Hg	100	Stable?
	90	Stable
	70	Stable
	50	Unstable

The yaw of the 100 percent Hg round was too small for a reliable analysis but it appeared to be stable, at least the yaw remained unmeasurably small throughout the firing range.

It is interesting to note that again the 50 percent filled rounds were unstable as predicted by Stewartson for this heavy liquid, assuming his theory can be applied to baffled rounds. However, for

these heavy liquids, his theory also predicts instability at 90 percent. With baffles, the 90 percent filled Hg round, however, was stable. Thus, a doubt is cast on the applicability of Stewartson's theory to baffled rounds, at least to the type of baffles used in these experiments. Perhaps at 50 percent fill, the baffles do not interfere with the fluid motion as much as at 90 percent. Therefore, Stewartson's theory may be valid at 50 percent fill and not at 90 percent fill.

8. SOME ENERGY CONSIDERATIONS

The fluid in a cylindrical cavity, rotating with the full spin of the shell, has an infinite number of free periods of oscillations. These frequencies depend on the geometry of the cavity, spin of the liquid, and the amount of liquid in the cavity or the percent of fill. The latter establishes the free boundary conditions. It has been shown by Stewartson that instabilities of the shell-liquid system occur whenever some frequency of the liquid coincides with the natural frequency of the solid shell. The rigid shell has two natural frequencies: precession and nutation. For the usual shell design, only the nutational frequency lies in the range of fluid frequencies. Hence, only the nutational mode can get into resonance with the fluid. In comparison with the whole spectrum of fluid frequencies, the nutational frequency is relatively low. Hence, only the lower modes of fluid frequenices are usually of importance. These modes, however, contain most of the energy of the disturbed fluid.

At resonance, the two oscillating systems, the shell and the fluid, can exchange energies very readily. The energy of the disturbed fluid motion is transferred to the yawing motion of the shell through work done by the pressure moment due to liquid.

In the transition phase, while the liquid is acquiring the spin, the situation is probably different. Mathematical solution of this problem is prohibitively difficult. Nevertheless, it appears plausible that, with a continuous gamut of spins present in the liquid during this regime, the spectrum of discrete frequencies of oscillations is absent. Therefore, the observed instabilities of the shell are not associated with resonance conditions as was the case in the steady state. Suitable transverse torques must still be present to keep the yaw growing, but, in the absence of the mathematical model, the origin of these torques is not clear.

It is interesting to note how relatively little of the energy of the shell-liquid system is contained in the yawing motion of the shell. For our 20mm 100 percent water filled shell, twice the kinetic energy contents in the various modes of the angular motions are, approximately, as follows. The figures are for the middle of the observed flight in the spark range.

Axial Spins	
Shell	400 joules
Liquid	10 joules
Yawing Motion (per sq. degree of yaw)	.02 joules
Dissipation	16 joules

To increase the yaw, therefore, requires relatively little of the fluid energy. As an example, to account for the observed decrease in the yaw damping rate for the 100 percent water filled shell (see Figure 5), it is necessary that the liquid moment be doing work at a rate of only about 4×10^{-4} watts. This is very small. Therefore, in order to predict such an effect theoretically would require a very precise knowledge of the fluid motion in the transition regime and this is not very promising.

9. SUMMARY

For nonspinning liquid filled shell both theory and experiment[6] agree that a liquid filler produces no detrimental effect on the dynamics of such a system. In fact, sloshing of the viscous liquid in a partially filled cavity should, by acting as an energy sink, increase the stability of the shell by increasing its yaw damping rate. Evidence of this has been observed in our experiments[6].

The situation is quite different for spin stabilized shell. Here, both theory and experiments agree that the liquid filler may have a very detrimental effect on the shell dynamics.

For a shell with a cylindrical cavity filled with a liquid, either partially or completely, with the liquid rotating with the full spin of the shell, Stewartson has shown[3] how to predict fill conditions at which the shell is dynamically unstable. The designer, therefore, may avoid such loadings.

However, because of the mathematical complexity of the problem, it was necessary for Stewartson to make the assumption that the fluid is inviscid yet rotating with the full spin of the shell. In reality, of course, it may take a relatively long time for the fluid to reach this state. Experiments with the 20mm shell have shown the existence of a broad zone of loading conditions at which the shell is dynamically unstable. This broad region of instability is probably associated with the unsteady phase of liquid motion in the process of reaching rigid body rotation. The time duration of this phase is shortened by high viscosity. For a completely filled cylindrical cavity, it is also shortened by the development of secondary flows within the cavity which markedly assist in the diffusion of vorticity throughout the liquid, thus expediting its approach to the rigid body rotation. For partially filled cavity, the direct experimental evidence on the effectiveness of secondary flows is not available. The indirect evidence, however, from the observed spin history of the partially filled shell, and their dynamic behavior when fired at longer ranges, suggest that the secondary flows continue to be effective, at least for higher percentages of fill.

Experiments with the 20mm shell also indicate that the rate of divergence of the nutational component of yaw increases with the increase in the specific gravity of the liquid. Thus, for thetrabromoethane, sp. gr. = 3, it is roughly three times as rapid as for water, sp. gr. = 1; and for mercury it is roughly thirteen times as fast. Thus, in spite of an effective assist from secondary flows in reaching the steady state, when Stewartson's theory can be used to predict shell dynamics, the transition time may still be long enough for prohibitively large yaws to develop. This has been demonstrated by long range firings with thetrabromoethane. With mercury, Plate 1 shows a similar result only at much shorter distance from the muzzle.

Therefore, it is safe to infer that for all liquids, with viscosity less than, say 500 c.s., and with specific gravity $\geq$ 1, it is desirable to suppress the transition phase by some sort of baffles. The effectiveness of baffles for heavier liquids has been demonstrated for liquid loadings in excess of 50 percent.

Whether the results obtained with the 20mm shell are scalable to larger shell remains to be determined. The effect of different geometries of the cavity also needs examination.

ACKNOWLEDGMENT

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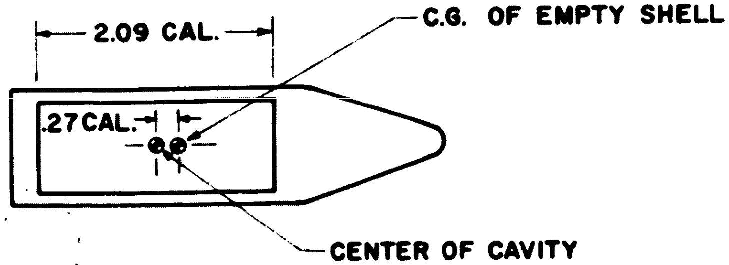
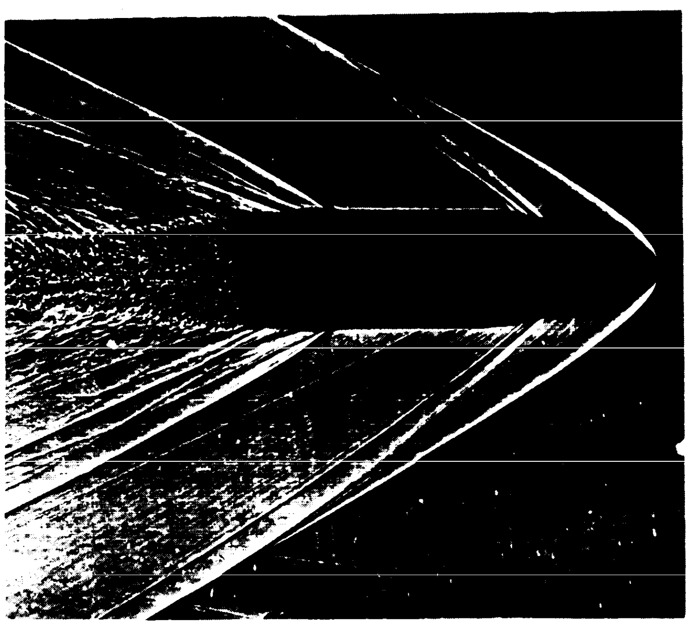
LIQUID FILLED SHELL



DISTANCES FROM THE MUZZLE

(M.V. = 2660 f.p.s.)

230 CALIBERS



4300 CALIBERS

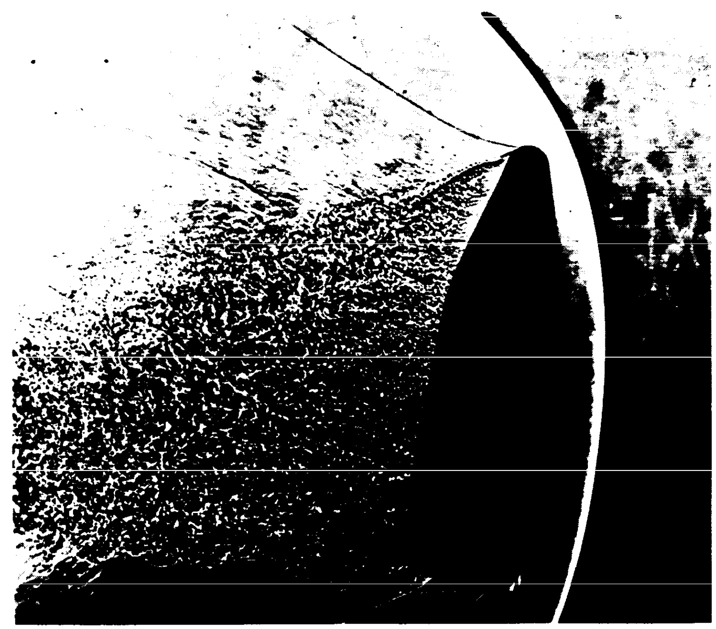
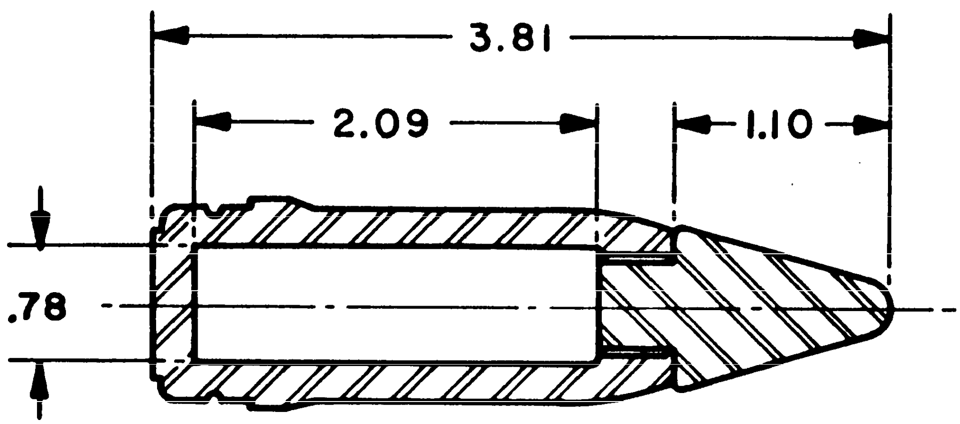


PLATE 1

20 MM SHELL, M56 (T282)



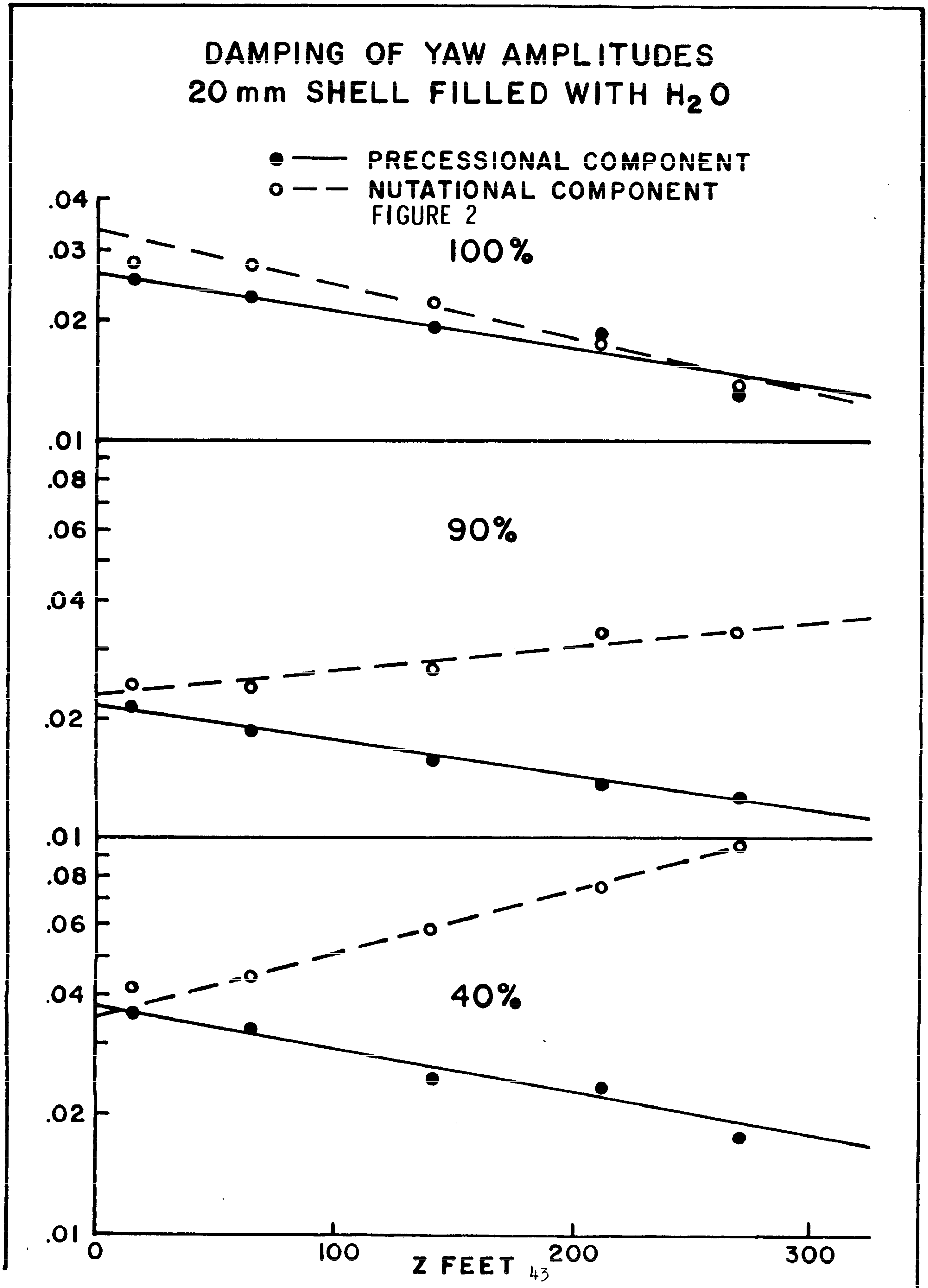
ALL DIMENSIONS IN CALIBERS

PHYSICAL CHARACTERISTICS EMPTY

WEIGHT, GRAMS	- 63	FROM BASE CAL.
A, GRAMS-CM^2	- 42.4	CENTER OF MASS - 1.49
B, GRAMS-CM^2	- 251.6	CENTER OF CAVITY - 1.22

CAVITY VOLUME 7.84 CU. CM.

FIGURE I


DAMPING OF YAW AMPLITUDES
20 mm SHELL FILLED WITH H_2O
PRECESSIONAL COMPONENT
NUTATIONAL COMPONENT
FIGURE 2
100%
90%
40%
.04
.03
.02
.01
.08
.06
.04
.02
.01
.08
.06
.04
.02
.01
0
100
200
300
Z FEET

EFFECT OF LIQUID (H_2O) ON "AERODYNAMIC COEFFICIENTS" VS. % OF FILL

FIGURE 3

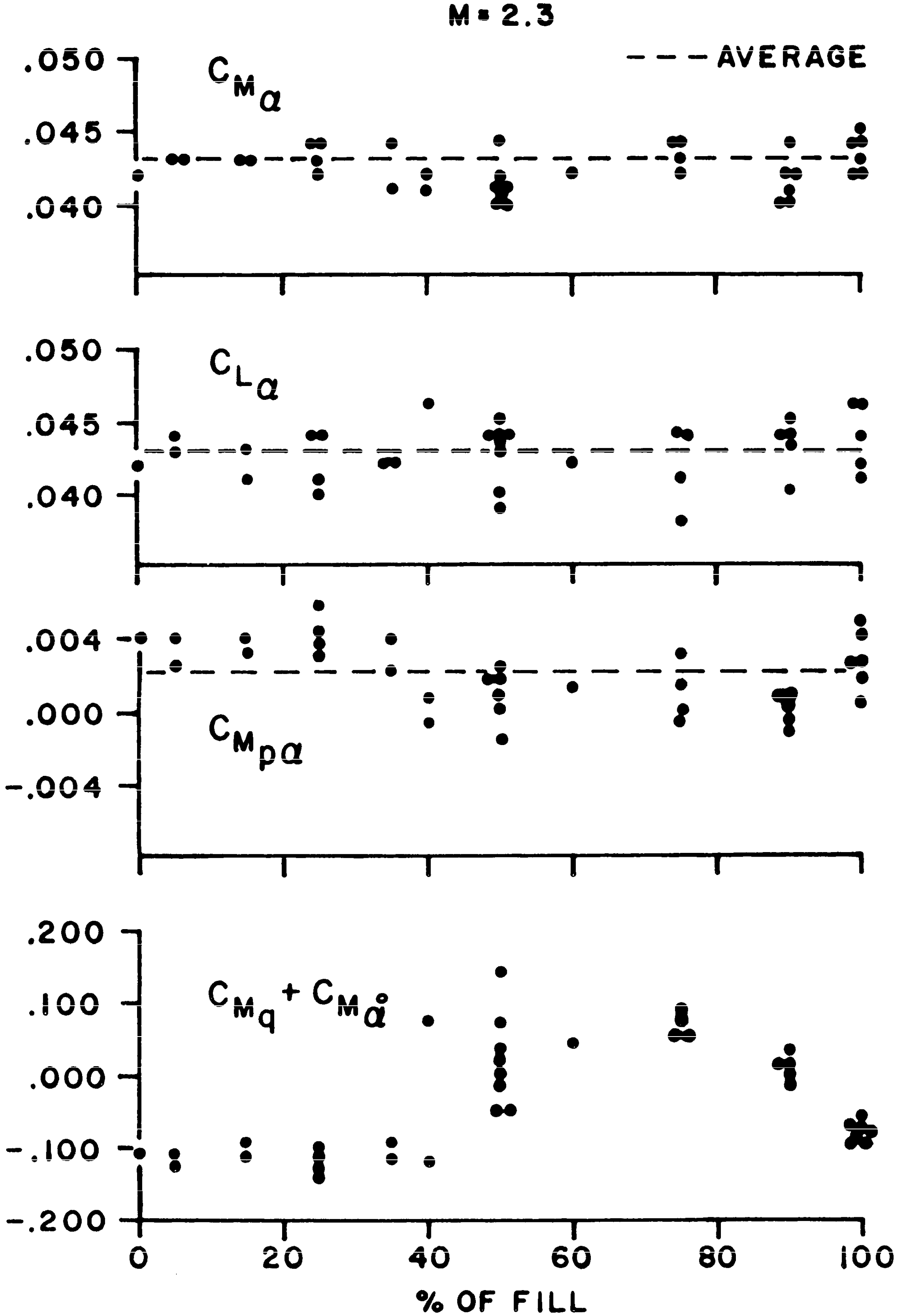


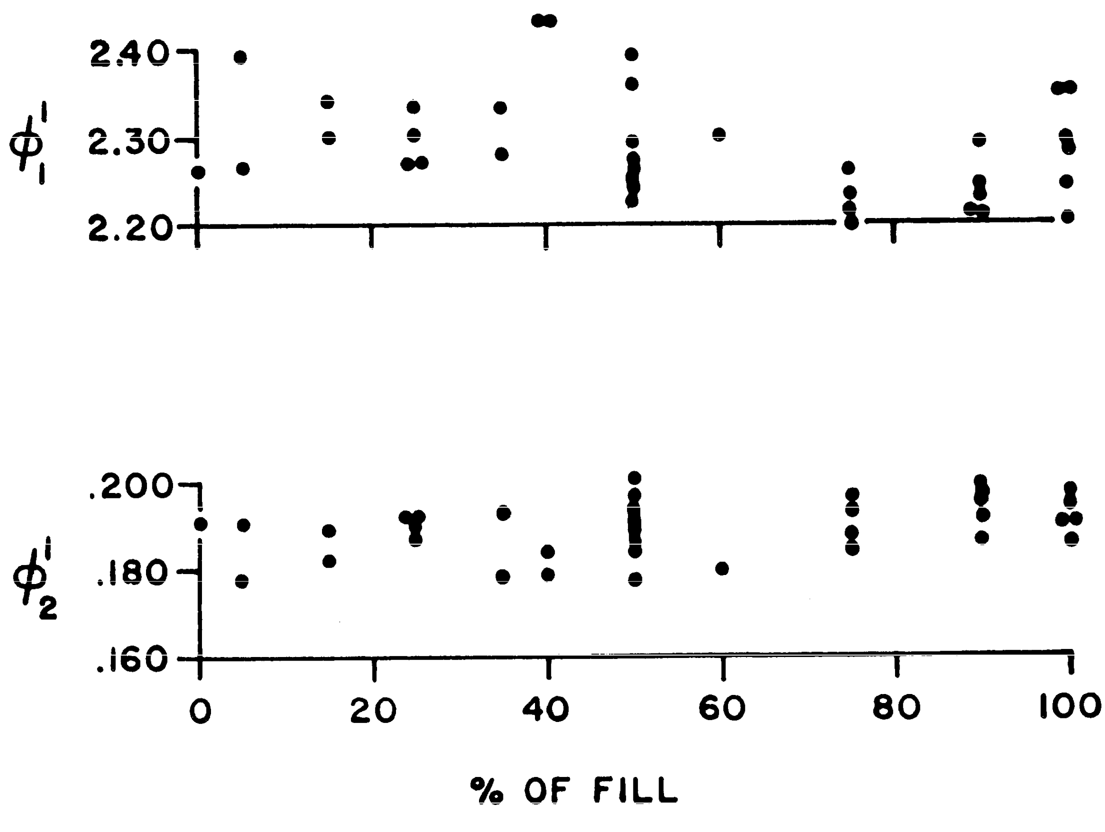
FIGURE 4

EFFECT OF LIQUID (H_2O) ON CIRCULAR RATES VS. % OF FILL

M = 2.3

ϕ_1' %CAL. NUTATIONAL

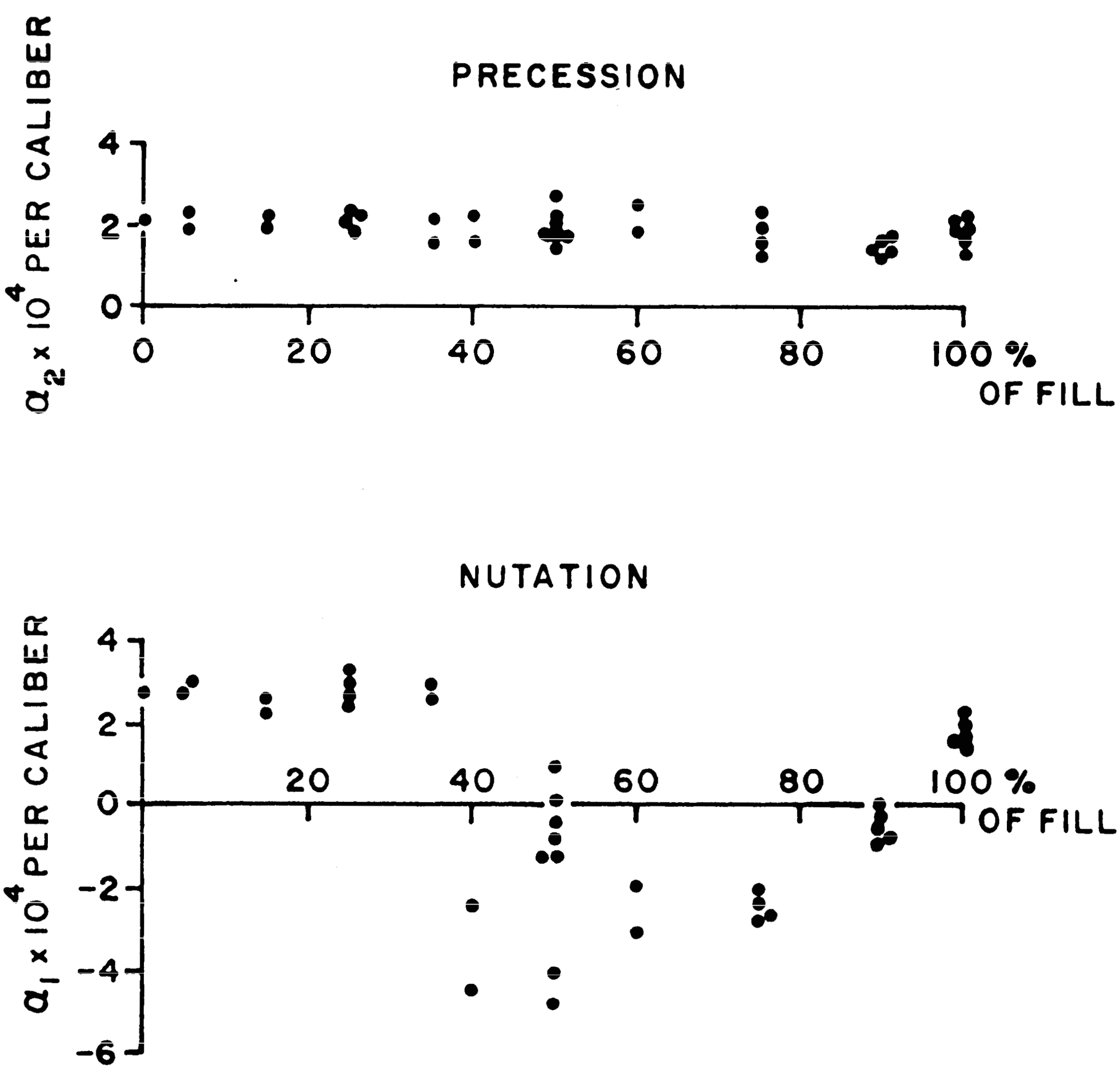
ϕ_2' %CAL. PRECESSIONAL



LIQUID (H_2O) FILLED SHELL

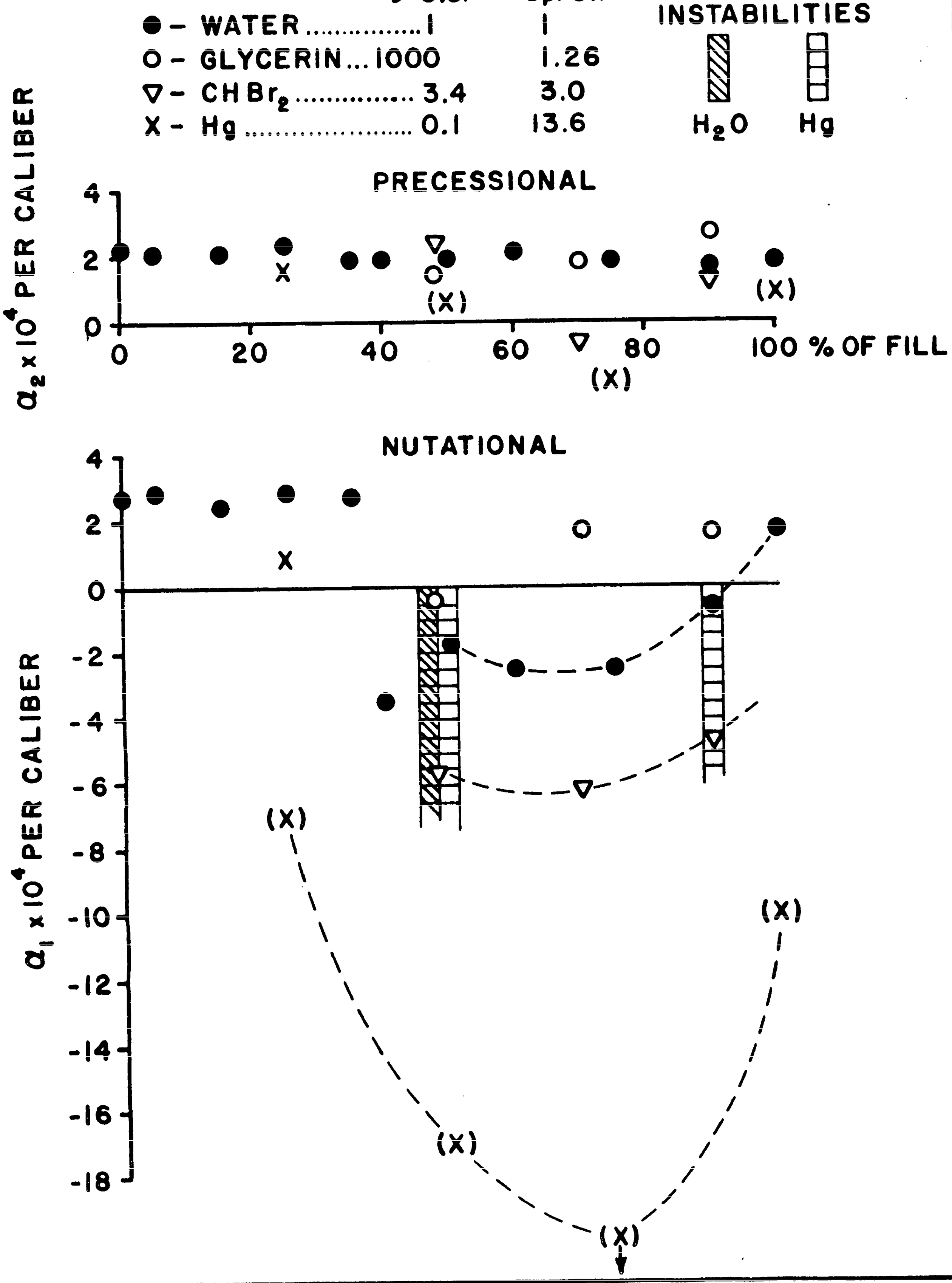
YAW DAMPING RATES, $e^{-\alpha_i p}$

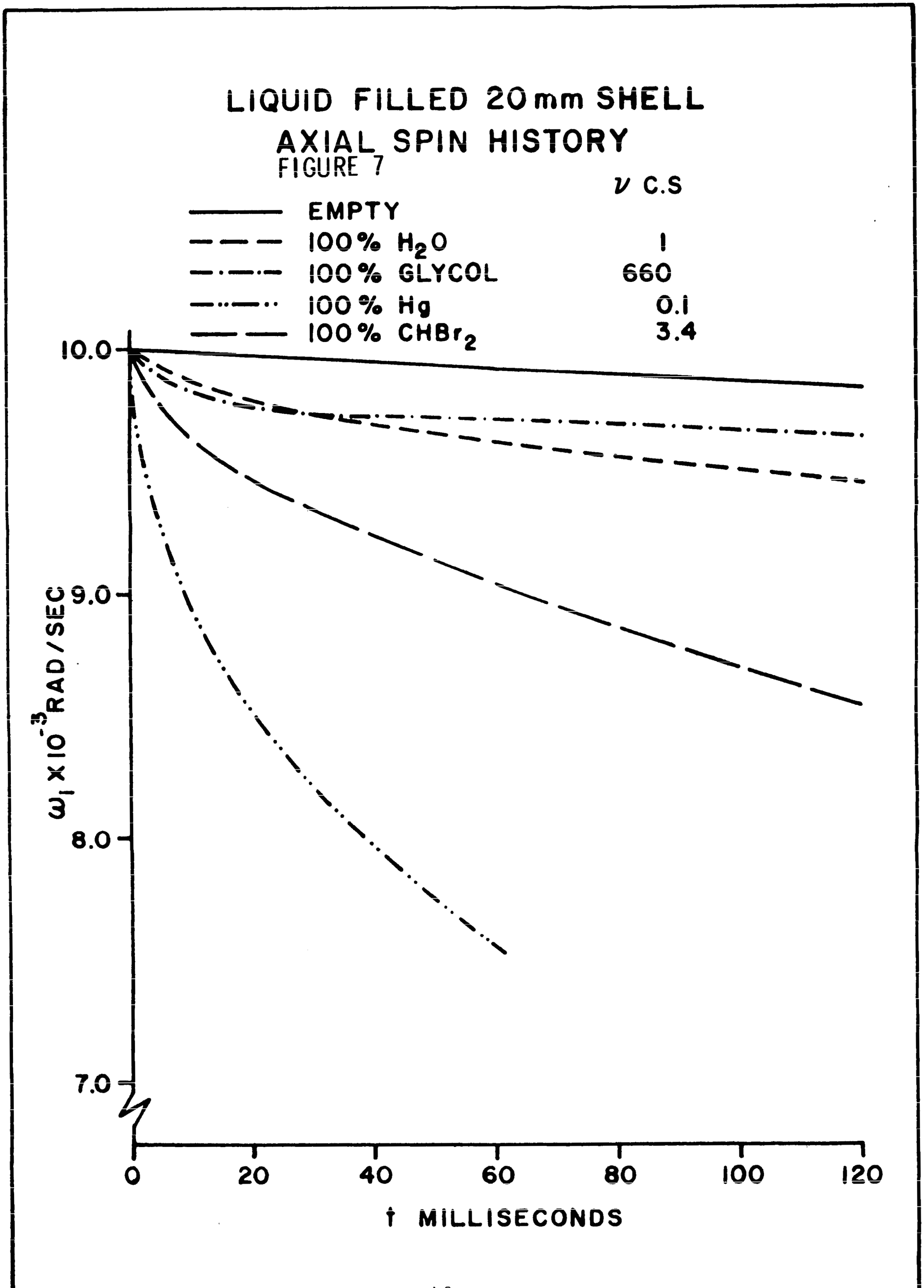
FIGURE 5



LIQUID FILLED SHELL
YAW DAMPING RATES, $e^{-\alpha_i p}$

FIGURE 6





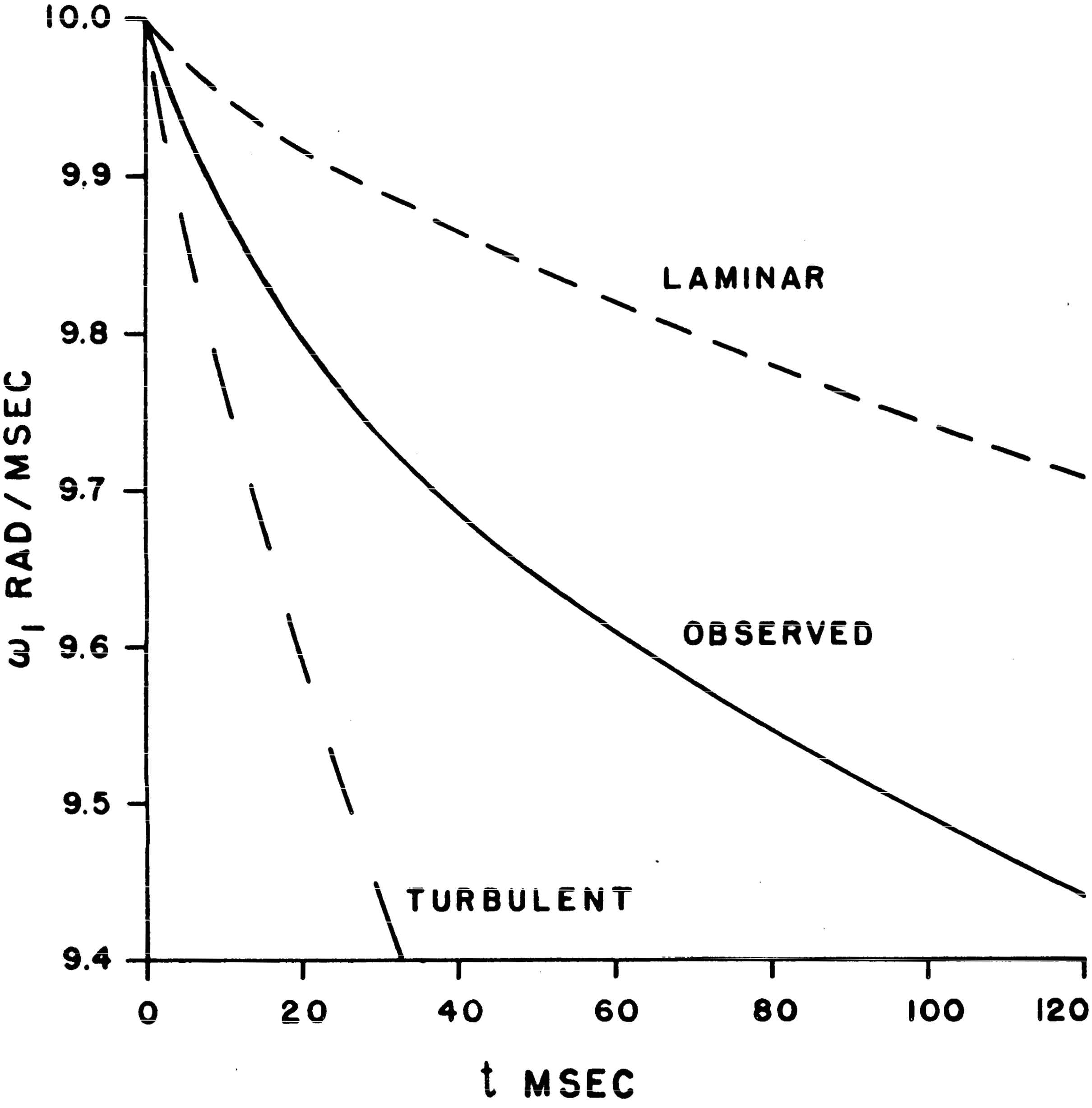
LIQUID FILLED 20 mm SHELL
AXIAL SPIN HISTORY
FIGURE 7
ν C.S
EMPTY
100% H_2O 1
100% GLYCOL 660
100% Hg 0.1
100% $CHBr_2$ 3.4
10.0
9.0
8.0
7.0
$\omega_1 \times 10^{-3}$ RAD/SEC
0
20
40
60
80
100
120
t MILLISECONDS

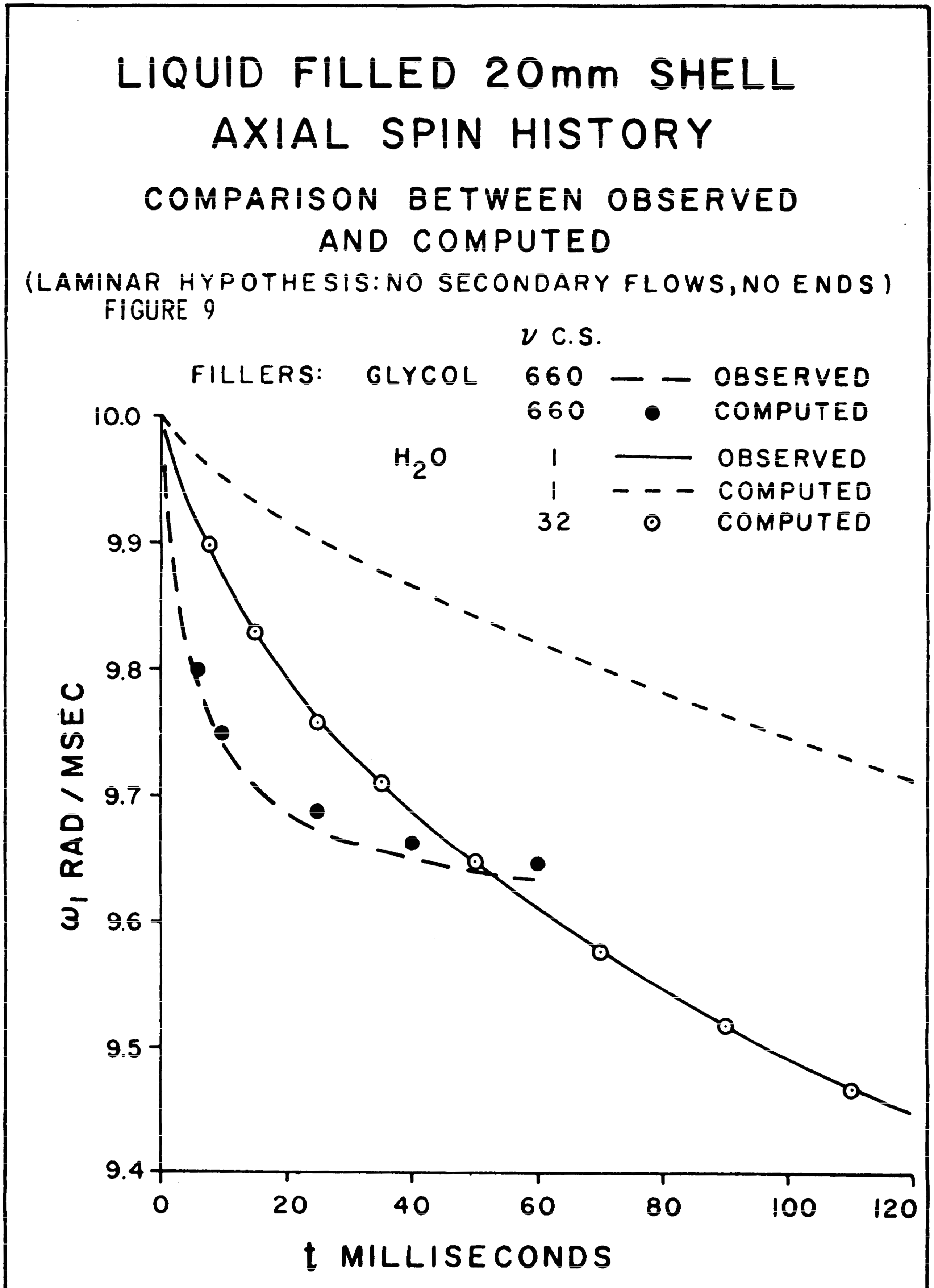
LIQUID FILLED 20mm SHELL

FIGURE 8

AXIAL SPIN HISTORY

100 % FULL (H_2O)





LIQUID FILLED 20mm SHELL
AXIAL SPIN HISTORY
COMPARISON BETWEEN OBSERVED AND COMPUTED
(LAMINAR HYPOTHESIS: NO SECONDARY FLOWS, NO ENDS)
FIGURE 9
ν C.S.
FILLERS: GLYCOL 660 OBSERVED
660 COMPUTED
H2O 1 OBSERVED
1 COMPUTED
32 COMPUTED
ω1 RAD / MSEC
10.0
9.9
9.8
9.7
9.6
9.5
9.4
0
20
40
60
80
100
120
t MILLISECONDS

PARTICLES TRAJECTORIES

LIQUID FILLED CYLINDER STARTED IMPULSIVELY

FIGURE 10 AT ω_0 = 3065 RPM

LIQUID = WATER + GLYCERIN

a= 1.88 cm ρ= 1.12 gms / cm^3 ν= .062 STOKES

POSITION OF PARTICLES AT 0.25 SECONDS INTERVALS

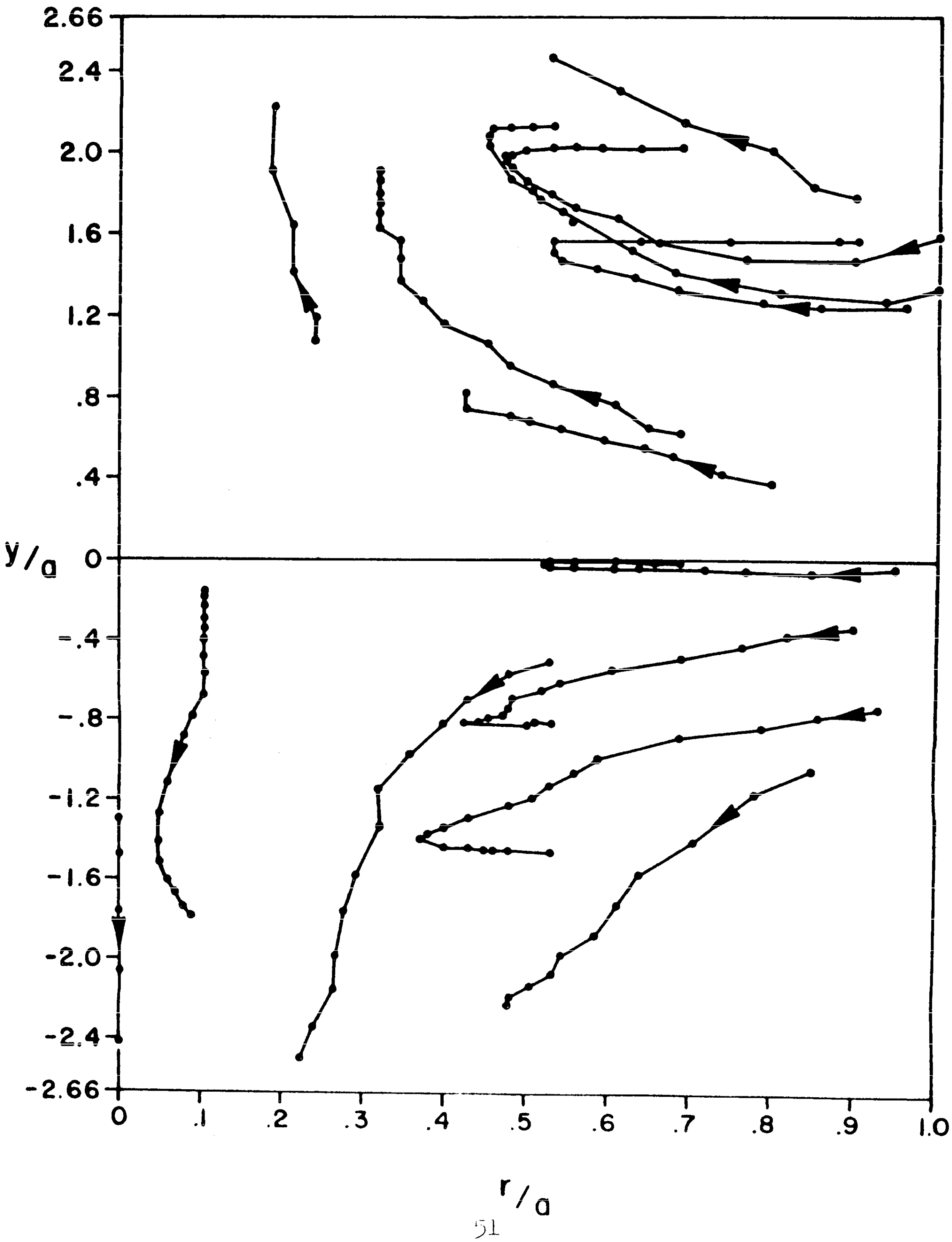
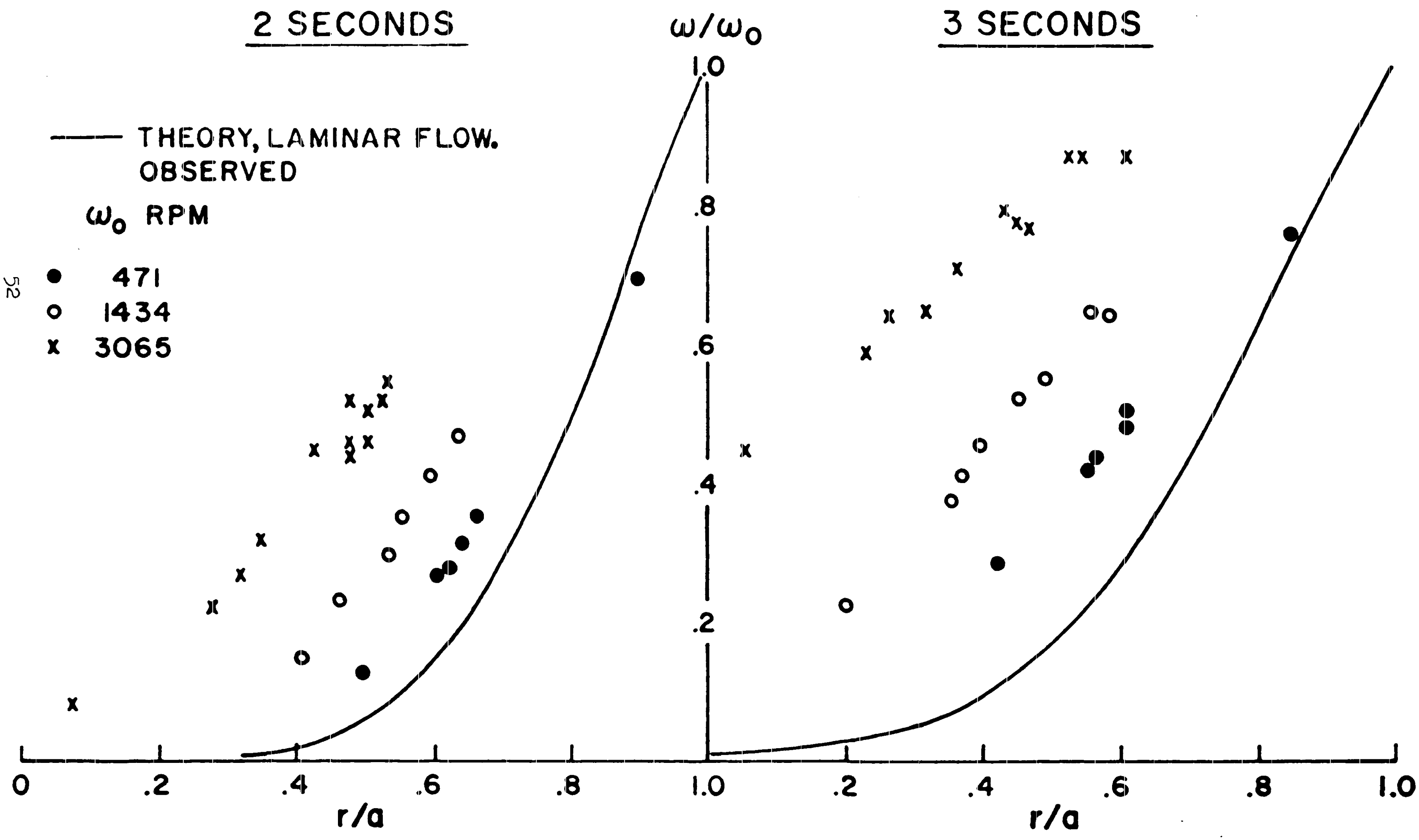


FIGURE II DISTRIBUTION OF VORTICITY WITHIN SPINNING CYLINDER STARTED IMPULSIVELY AT ω_0



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Bodies of revolution - Stability

Liquid-filled shell- Stability

Experiments with a 20mm M56 shell, whose cylindrical cavity, fineness ratio 2.7, was filled with different amounts of various liquids, demonstrated the existence of dynamic instability for a broad range of loading conditions. It was found that, for water-like liquids, the nutational component of yaw is divergent when 40 percent to 90 percent of the cavity is filled. For mercury, the yaw is divergent from about 25 percent to 100 percent. The severity of instability or the rate of divergence of yaw increases with the increase in the specific gravity of the liquid.

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